1.4 삼각함수

예제1)

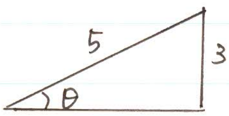
① $90^\circ = \frac{\pi}{2}$ ② $-30^\circ = -\frac{\pi}{6}$ ③ $\frac{2}{3}\pi = 120^\circ$

* $360^\circ = 2\pi$, $\pi = 180^\circ$

나타내려는 쪽을 분자, 현재상태를 분모로 하는 계수를 곱한다.

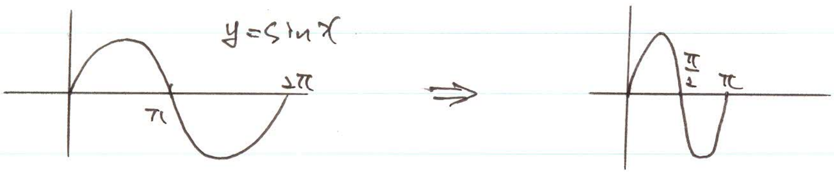
④ $2 = 2 \times \frac{180^\circ}{\pi} = \frac{360^\circ}{\pi}$ (수학) $\fallingdotseq 114.6^\circ$ (공학)

예제2) $\sin\theta = \frac{3}{5}$ (단, $\frac{\pi}{2} < \theta < \pi$)



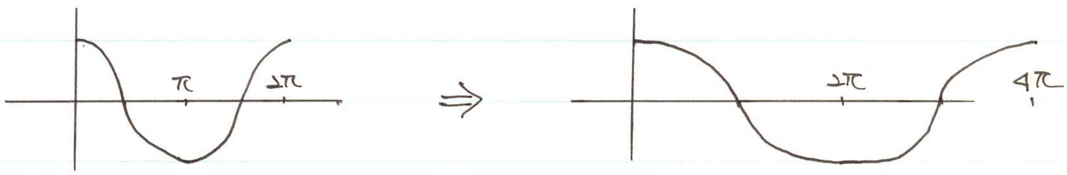
① $\cos\theta = \frac{4}{5}$ ② $\tan\theta = \frac{3}{4}$ ③ $\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

예제3) $y = \sin 2x$: $y = \sin x$ 를 x축으로 $\frac{1}{2}$ 축소.



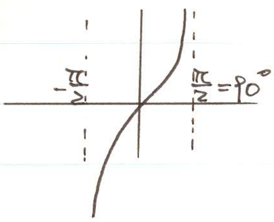
∴ 주기는 π

예제4) $y = \cos\frac{1}{2}x$: $y = \cos x$ 를 x축으로 2배 확대.

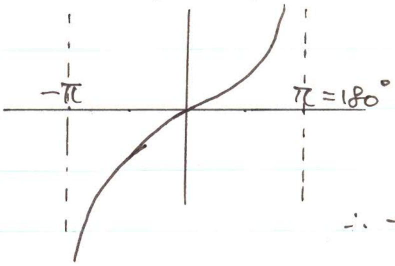


∴ 주기는 4π

예제5) $y = \tan \frac{1}{2}x$: $y = \tan x$ 를 x축으로 2배 확대.

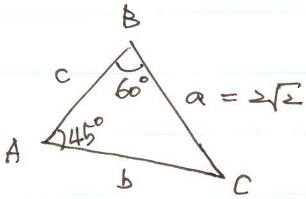


$\Rightarrow$



∴ 주기는 2π

예제6)



b, $R = ?$

① $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{2\sqrt{2}}{\sin 45°} = \frac{b}{\sin 60°}$

$\frac{2\sqrt{2}}{1/\sqrt{2}} = 4 = \frac{b}{\sqrt{3}/2}$

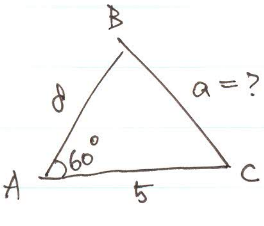
$\therefore b = 2\sqrt{3}$

② $\frac{a}{\sin A} = 2R$

$\frac{2\sqrt{2}}{\sin 45°} = 4 = 2R$

$\therefore R = 2$

예제7)



$a^2 = b^2 + c^2 - 2bc\cos A$

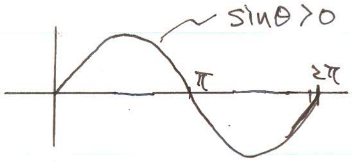
$= 5^2 + 8^2 - 2 \cdot 5 \cdot 8 \cdot \cos 60°$

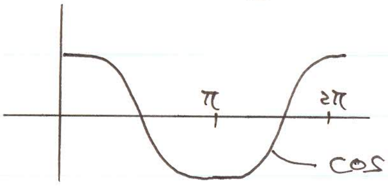
$= 25 + 64 - 80 \times \frac{1}{2} \quad = 49$

$\therefore a = 7$

1. $\sin\theta > 0$, $\cos\theta < 0$ 인 $\theta = ?$

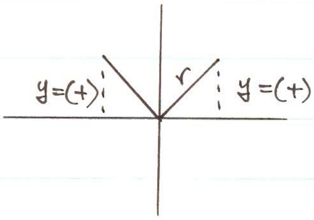
$\sin\theta > 0 \rightarrow$ 1, 2사분면

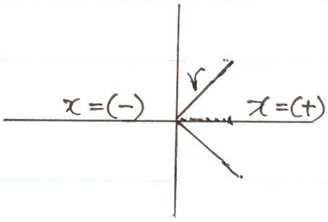




$\cos\theta < 0 \rightarrow$ 2, 3사분면

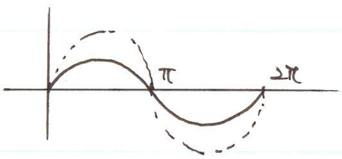
∴ θ는 2사분면





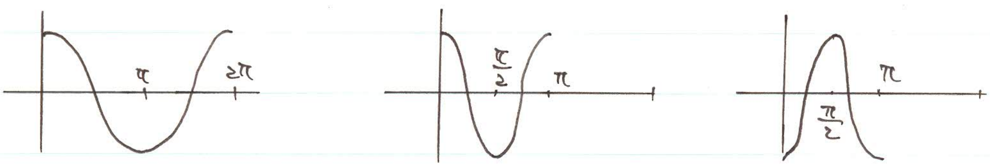
2.

① $y = \frac{1}{2}\sin x$: $y = \sin x$를 y축으로 $\frac{1}{2}$ 축소.



∴ 주기는 2π

② $y = -\cos 2x$: $y = \cos x$를 x축으로 $\frac{1}{2}$ 축소 x축에 대하여 대칭

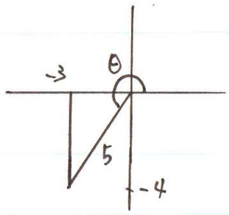


∴ 주기는 π

Date No

3. $\sin\theta = -\frac{4}{5}$, θ는 3사분면의 각일때 $\cos\theta,\ \tan\theta = ?$

① Method 1



$\cos\theta = \frac{-3}{5}$

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$

② Method 2

$$\sin^2\theta + \cos^2\theta = 1$$
$$\frac{16}{25} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{9}{25}$$
$$\cos\theta = \pm\frac{3}{5}$$

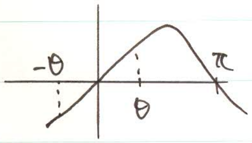
θ가 3사분면이므로 $\cos\theta = -\frac{3}{5}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$$

4.

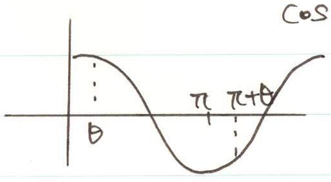
① $\sin\left(-\frac{\pi}{6}\right) = \sin(-30^\circ) \underset{\uparrow}{=} -\sin 30^\circ = -\frac{1}{2}$

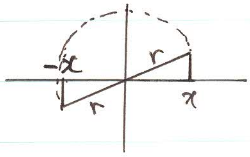
$\sin(-\theta) = -\sin\theta$



② $\cos 210^\circ = \cos(180^\circ + 30^\circ) \underset{\uparrow}{=} -\cos 30^\circ$

$\cos(\pi+\theta) = -\cos\theta$





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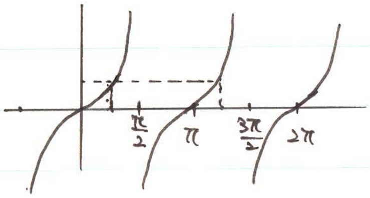
③ $\tan \frac{5}{4}\pi = \tan\left(\pi + \frac{1}{4}\pi\right) = \tan \frac{1}{4}\pi = 1$

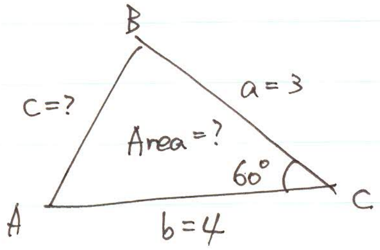
$\tan(\pi+\theta) = \tan\theta$



④ $\sin 690^\circ = \sin(720^\circ - 30^\circ) = \sin(-30^\circ) = -\sin 30^\circ = -\frac{1}{2}$

$\sin(2n\pi+\theta) = \sin\theta$

5.



$$c^2 = a^2+b^2-2ab\cos C$$
$$= 3^2+4^2-2\times3\times4\times\cos 60^\circ$$
$$= 9+16-24\times\frac{1}{2} = 13$$
$$\therefore\ c = \sqrt{13}$$

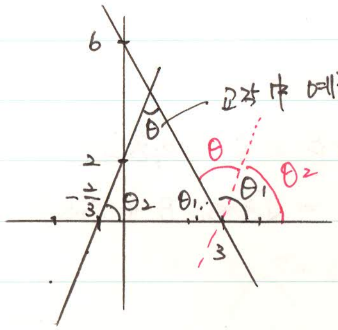
$$S = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}\times3\times4\times\underbrace{\sin 60^\circ}\ \frac{\sqrt{3}}{2}$$
$$= 3\sqrt{3}$$

1.5 삼각함수의 덧셈정리와 여러가지 공식

예제1) 두 직선 $2x+y-6=0$, $3x-y+2=0$ 이 이루는 예각 θ의 크기는?

sol) 두 직선을 정리하면

$$\begin{cases} y = -2x+6 \\ y = 3x+2 \end{cases}$$



• 둔각 θ_1을 선택하면

$\theta = \theta_1 - \theta_2$ 이고.

$\tan\theta_1 = \frac{6}{-3} = -2$, $\tan\theta_2 = 3$ 이므로.

직선의 기울기와 동일함.

$$\tan\theta = \tan(\theta_1 - \theta_2)$$

$$= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \cdot \tan\theta_2}$$

$$= \frac{-2-3}{1+(-2)\cdot 3} = \frac{-5}{-5} = 1$$

$$\therefore \theta = \frac{\pi}{4}$$

• 예각 θ_1을 선택하면

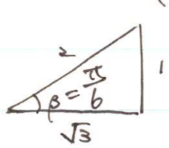
$\theta_1 + \theta_2 + \theta = 180^\circ$ 또는 $\theta = 180^\circ - (\theta_1 + \theta_2)$로 부터 시작.

예제2) $\sqrt{3}\sin\theta + \cos\theta$ 를 $r\cdot\sin(\theta+\alpha)$ 꼴로 나타내어라

sol) • $\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$ 를 이용하면

$\sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta = \sqrt{3}\sin\theta + \cos\theta$ 라면

$$\left.\begin{aligned} \alpha &= \theta \\ \cos\beta &= \sqrt{3} \\ \sin\beta &= 1 \end{aligned}\right\}$$ 로 부터



$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

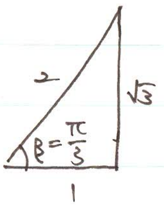
$$\therefore \begin{cases} 2\cos\frac{\pi}{6} = \sqrt{3} \\ 2\sin\frac{\pi}{6} = 1 \end{cases}$$

$$\begin{aligned}\therefore \sqrt{3}\cdot\sin\theta + \cos\theta &= \sin\theta\cdot 2\cdot\cos\frac{\pi}{6} + \cos\theta\cdot 2\cdot\sin\frac{\pi}{6} \\ &= 2\left(\sin\theta\cdot\cos\frac{\pi}{6} + \cos\theta\cdot\sin\frac{\pi}{6}\right) \\ &= 2\cdot\sin\left(\theta + \frac{\pi}{6}\right)\end{aligned}$$

- $\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$ 를 이용하면

$\cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta = \sqrt{3}\sin\theta + \cos\theta$ 라면

$$\left.\begin{array}{l}\alpha = \theta. \\ \sin\beta = \sqrt{3} \\ \cos\beta = 1\end{array}\right\}$$ 로 부터



$\sin\beta = \frac{\sqrt{3}}{2}$

$\cos\beta = \frac{1}{2}$

$$\therefore \begin{cases} 2\sin\beta = \sqrt{3} \\ 2\cos\beta = 1 \end{cases}$$

$$\begin{aligned}\therefore \sqrt{3}\sin\theta + \cos\theta &= 2\sin\frac{\pi}{3}\cdot\sin\theta + 2\cdot\cos\frac{\pi}{3}\cdot\cos\theta \\ &= 2\left(\cos\theta\cdot\cos\frac{\pi}{3} + \sin\theta\cdot\sin\frac{\pi}{3}\right) \\ &= 2\cdot\cos\left(\theta - \frac{\pi}{3}\right)\end{aligned}$$

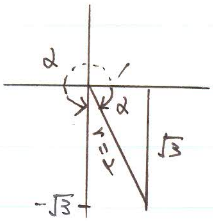
예제 3) $y = \sin x - \sqrt{3}\cdot\cos x + 1$ 일때 y의 범위를 구하시오.

sol) 삼각함수 합성 공식을 이용하면

$$\begin{aligned}\sin x - \sqrt{3}\cdot\cos x &= \sqrt{1^2 + (-\sqrt{3})^2}\cdot\sin(\theta+\alpha) \\ &= 2\cdot\sin(\theta+\alpha)\end{aligned}$$ 이고.

$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}} = \frac{1}{2}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}} = \frac{-\sqrt{3}}{2}$

cos 하면(+), sin 하면(−) 이므로 α는 4 사분면

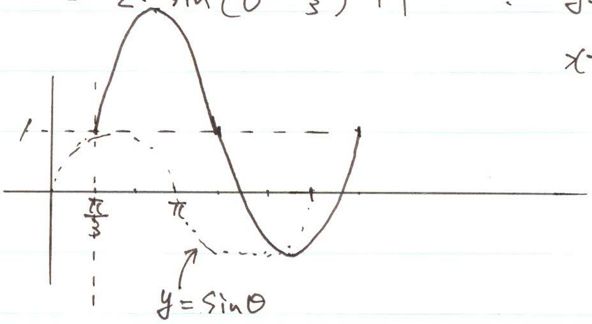


$\therefore \alpha = \frac{5}{3}\pi$ or $-\frac{1}{3}\pi$

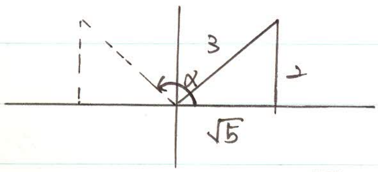
$\therefore \sin x - \sqrt{3}\cdot\cos x = 2\cdot\sin\left(\theta - \frac{\pi}{3}\right)$

$\therefore y = \sin x - \sqrt{3}\cos x + 1$

$= 2\cdot\sin\left(\theta - \frac{\pi}{3}\right) + 1$: $y = \sin\theta$를 y축으로 2배 확대
x축으로 $+\frac{\pi}{3}$, y축으로 +1 이동



예제4) $\sin\alpha = \frac{2}{3}$, (단, $\frac{\pi}{2} < \alpha < \pi$) 일때 다음 값을 구하라.



단, 조건에서 α는 2사분면 각이므로

∴ 그림에서 $\cos\alpha = \frac{-\sqrt{5}}{3}$

또는 $\sin^2\alpha + \cos^2\alpha = 1$ 로부터

$\cos^2\alpha = 1 - \frac{4}{9} = \frac{5}{9}$

$\cos\alpha = \pm\frac{\sqrt{5}}{3}$ 이고 조건에서 α가 2사분면이면 cosα는(−)

$\therefore \cos\alpha = -\frac{\sqrt{5}}{3}$

① $\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha = 2\cdot\frac{2}{3}\cdot\left(-\frac{\sqrt{5}}{3}\right) = -\frac{4\sqrt{5}}{9}$

② $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(-\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$

③ $\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{-\frac{4\sqrt{5}}{9}}{\frac{1}{9}} = -4\sqrt{5}$

예제5) $\cos\alpha = -\frac{3}{4}$, (단, $\frac{\pi}{2} < \alpha < \pi$) 일때 다음을 구하라.

sol) $\sin^2\alpha + \cos^2\alpha = 1$ 로부터

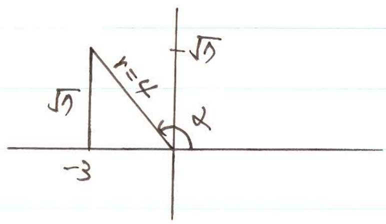
$\sin^2\alpha = 1 - \left(-\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$

$\sin\alpha = \pm\frac{\sqrt{7}}{4}$

단. 조건으로부터 α는 2사분면 각이고, $\sin\alpha$는 (+)

$\therefore \sin\alpha = \frac{\sqrt{7}}{4}$

또는



$\therefore \sin\alpha = \frac{\sqrt{7}}{4}$

① $\sin\frac{\alpha}{2} = ?$

반각의 공식을 이용하면

$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{1-\left(-\frac{3}{4}\right)}{2} = \frac{\frac{7}{4}}{2} = \frac{7}{8}$

$\therefore \sin\frac{\alpha}{2} = \pm\sqrt{\frac{7}{8}} = \pm\frac{\sqrt{14}}{4}$

단 조건에서 $\frac{\pi}{2} < \alpha < \pi$ 이므로 $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$

즉 $\frac{\alpha}{2}$는 1사분면 각이므로

$\sin\frac{\alpha}{2} = \frac{\sqrt{14}}{4}$

② $\cos\frac{\alpha}{2} = ?$

반각의 공식을 이용하면

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} = \frac{1+\left(-\frac{3}{4}\right)}{2} = \frac{\frac{1}{4}}{2} = \frac{1}{8}$$

$$\therefore \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1}{8}} = \pm\frac{\sqrt{2}}{4}$$

단 조건에서 $\frac{\pi}{2} < \alpha < \pi$ 이므로 $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$

즉 $\frac{\alpha}{2}$는 1사분면 각이므로

$$\cos\frac{\alpha}{2} = +\frac{\sqrt{2}}{4}$$

예제 6) $\sin\alpha\cdot\cos\beta = \frac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\}$

① $\sin 75^\circ\cdot\cos 15^\circ = \frac{1}{2}\{\sin(75^\circ+15^\circ)+\sin(75^\circ-15^\circ)\}$

$= \frac{1}{2}(\sin 90^\circ + \sin 60^\circ)$

$= \frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\right)$

$= \frac{2+\sqrt{3}}{4}$

② $\cos\alpha\cdot\cos\beta = \frac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\}$

$\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ = \frac{1}{2}(\cos 60^\circ + \cos(-20^\circ))\cdot\cos 80^\circ$

$= \frac{1}{2}\left(\frac{1}{2}\cdot\cos 80^\circ + \cos 20^\circ\cdot\cos 80^\circ\right)$

$= \frac{1}{4}\cos 80^\circ + \frac{1}{4}(\cos 100^\circ + \cos(-60^\circ))$

$= \frac{1}{4}\left(\cos 80^\circ + \cos 100^\circ + \frac{1}{2}\right)$

여기서, $\cos 100^\circ = \cos(180^\circ - 80^\circ)$; cos 덧셈정리에 의하여

$= \cos 180\cdot\cos 80^\circ + \sin 180^\circ\cdot\sin 80^\circ$

$= -1\cdot\cos 80^\circ + 0\cdot\sin 80^\circ$

$= -\cos 80^\circ$

$\therefore \cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ = \frac{1}{4}\left(\cos 80^\circ - \cos 80^\circ + \frac{1}{2}\right) = \frac{1}{8}$

1. 삼각함수의 덧셈정리.

① $\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

② $\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

③ $\tan 105^\circ = \tan(60^\circ + 45^\circ)$

$= \dfrac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \cdot \tan 45^\circ}$

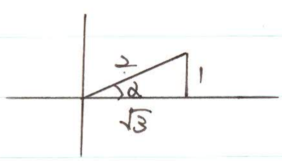
$= \dfrac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} \times \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$

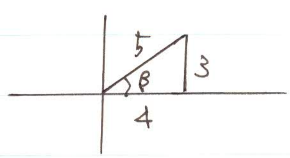
$= \dfrac{(1+\sqrt{3})^2}{1-3} = \dfrac{1+2\sqrt{3}+3}{-2} = \dfrac{4+2\sqrt{3}}{-2} = -(2+\sqrt{3})$

덧셈정리.

2. $\sin\alpha = \frac{1}{2}$, $\cos\beta = \frac{4}{5}$ 단, α, β는 1사분면의 각.

① $\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$





$\therefore \cos\alpha = \frac{\sqrt{3}}{2}$ $\qquad \sin\beta = \frac{3}{5}$

$= \frac{1}{2} \cdot \frac{4}{5} + \frac{\sqrt{3}}{2} \cdot \frac{3}{5}$

$= \frac{4+3\sqrt{3}}{10}$

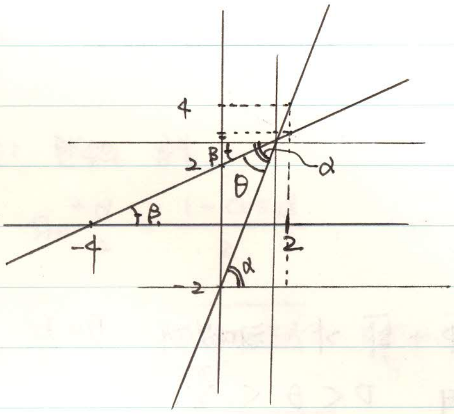
② $\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$

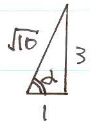
$= \frac{\sqrt{3}}{2}\cdot\frac{4}{5} + \frac{1}{2}\cdot\frac{3}{5}$

$= \frac{4\sqrt{3}+3}{10}$

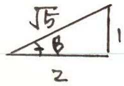
3. 두 직선이 이루는 예각 θ의 크기는?

$$\left.\begin{array}{l} 3x - y - 2 = 0 \\ x - 2y + 4 = 0 \end{array}\right\} \rightarrow \begin{array}{l} y = 3x - 2 \\ y = \frac{1}{2}x + 2 \end{array}$$





$\therefore \sin\alpha = \frac{3}{\sqrt{10}}, \quad \cos\alpha = \frac{1}{\sqrt{10}}$



$\therefore \sin\beta = \frac{1}{\sqrt{5}}, \quad \cos\beta = \frac{2}{\sqrt{5}}$

그림에서 $\theta = \alpha - \beta$

$\sin\theta = \sin(\alpha-\beta)$

$= \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$

$= \frac{3}{\sqrt{10}}\cdot\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}}\cdot\frac{1}{\sqrt{5}}$

$= \frac{6\sqrt{2}}{10} - \frac{\sqrt{2}}{10}$

$= \frac{5\sqrt{2}}{10}$

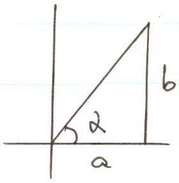
$= \frac{\sqrt{2}}{2} \left(= \frac{1}{\sqrt{2}}\right)$ $\qquad \therefore \theta = 45^\circ$

4. 최대·최소 구하기 (삼각함수의 합성)

NOTE: $f(x) = a\cdot\sin\theta + b\cdot\cos\theta$

$= \sqrt{a^2+b^2}\cdot\sin(\theta+\alpha)$

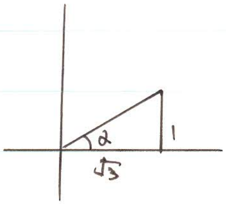
단, $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}, \quad \sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$



① $f(x) = \sqrt{3} \cdot \sin x + \cos x$

$= \sqrt{3+1} \cdot \sin(x+\alpha)$

$= 2 \cdot \sin(x+\alpha)$



$\alpha = \frac{\pi}{6}$

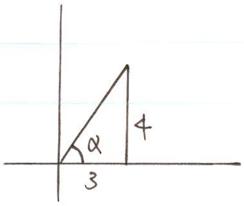
$\therefore f(x).\max = 2$

$f(x).\min = -2$

② $g(x) = 3\sin x + 4\cos x - 2$

$= \sqrt{9+16} \cdot \sin(x+\alpha) - 2$

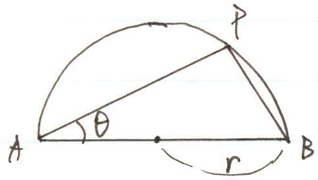
$= 5 \cdot \sin(x+\alpha) - 2$



$\therefore g(x).\max = 3$

$g(x).\min = -7$

5. 삼각함수 합성.



$\sqrt{3} \cdot \overline{AP} + \overline{BP}$ 가 최대일때 $\theta = ?$

그림에서 $0 < \theta < \frac{\pi}{2}$

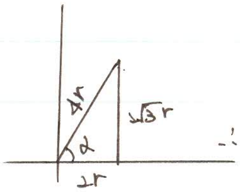
$\overline{AP} = 2 \cdot r \cdot \cos\theta \cdot$

$\overline{BP} = 2 \cdot r \cdot \sin\theta$

$\therefore \sqrt{3}\,\overline{AP} + \overline{BP} = 2\sqrt{3}r \cdot \cos\theta + 2r\sin\theta$

$= \sqrt{4r^2 + 12r^2} \cdot \sin(\theta+\alpha)$

$= 4r \cdot \sin(\theta+\alpha)$



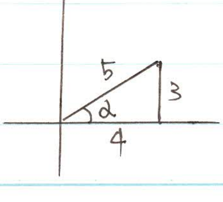
$\therefore \alpha = \frac{\pi}{3}$

$\therefore \sqrt{3}\,\overline{AP} + \overline{BP}$ 가 최대이기 위하여 $\sin(\theta+\alpha) = 1$

$\theta + \frac{\pi}{3} = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

6. $\sin\alpha = \frac{3}{5}$ (단, $0 < \alpha < \frac{\pi}{2}$)



$\cos\alpha = \frac{4}{5}$

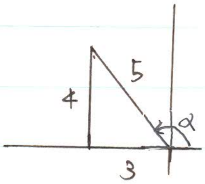
NOTE: 배각의 공식.

$$\sin 2\alpha = \sin(\alpha+\alpha) = \sin\alpha\cdot\cos\alpha + \cos\alpha\cdot\sin\alpha$$
$$= 2\sin\alpha\cdot\cos\alpha.$$
$$\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\,\cos\alpha - \sin\alpha\cdot\sin\alpha$$
$$= \cos^2\alpha - \sin^2\alpha$$

① $\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$

② $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

2. $\sin\alpha = \frac{4}{5}$ (단, $\frac{\pi}{2} < \alpha < \pi$)



$\cos\alpha = \frac{-3}{5}$

NOTE: 반각의 공식.

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2},\quad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2},\quad \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

① $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$ ($\because \frac{\pi}{2} < \alpha < \pi$ 이므로 $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$ $\therefore \sin\frac{\alpha}{2}$는 $(+)$)

$$= \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}$$
$$= \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}}$$

② $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$ ($\because \frac{\pi}{2} < \alpha < \pi$ 이므로 $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$ $\therefore \cos\frac{\alpha}{2}$는 $(+)$)

$$= \sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}$$
$$= \sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{2}{10}} = \frac{1}{\sqrt{5}}$$

8. 곱 → 합 or 차

① $\sin 37.5^\circ \cdot \sin 7.5^\circ = -\frac{1}{2}\left\{\cos(37.5^\circ + 7.5^\circ) - \cos(37.5^\circ - 7.5^\circ)\right\}$

$$= -\frac{1}{2}\left(\cos 45^\circ - \cos 30^\circ\right)$$

$$= -\frac{1}{2}\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\right)$$

$$= -\frac{1}{2} \cdot \frac{\sqrt{2}-\sqrt{3}}{2} = \frac{\sqrt{3}-\sqrt{2}}{4}$$

② $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ$

$$= -\frac{1}{2}\left\{\cos(20^\circ+40^\circ) - \cos(20^\circ-40^\circ)\right\}\sin 80^\circ$$

$$= -\frac{1}{2}\left(\cos 60^\circ - \cos 20^\circ\right)\cdot \sin 80^\circ$$

$$= -\frac{1}{4}\sin 80^\circ + \frac{1}{2}\cos 20^\circ \cdot \sin 80^\circ$$

$$= -\frac{1}{4}\sin 80^\circ + \frac{1}{4}\left\{\sin(20^\circ+80^\circ) - \sin(20^\circ-80^\circ)\right\}$$

$$= -\frac{1}{4}\sin 80^\circ + \frac{1}{4}\left\{\sin 100^\circ + \sin 60^\circ\right\}$$

$$= -\frac{1}{4}\sin 80^\circ + \frac{1}{4}\cdot \sin 100^\circ + \frac{\sqrt{3}}{8}$$

where; $\sin 100^\circ = \sin(180^\circ - 80^\circ) = \sin 180^\circ \cos 80^\circ - \cos 180^\circ \cdot \sin 80^\circ$

$$= 0 - (-1)\cdot \sin 80^\circ$$

$$= \sin 80^\circ$$

$\therefore \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ = \frac{\sqrt{3}}{8}$

9. 합 or 차 → 곱.

① $\cos 75^\circ - \cos 15^\circ = -2 \cdot \sin \frac{75^\circ + 15^\circ}{2} \cdot \sin \frac{75^\circ - 15^\circ}{2}$

$= -2 \cdot \sin 45^\circ \cdot \sin 30^\circ$

$= -2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = -\frac{1}{\sqrt{2}}$

② $\sin 130^\circ - \sin 110^\circ + \sin 10^\circ = \sin 50^\circ - \sin 70^\circ + \sin 10^\circ$

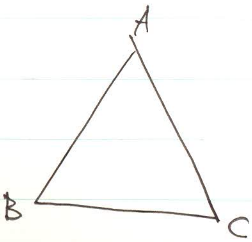
$= 2 \cdot \cos \frac{50^\circ + 70^\circ}{2} \cdot \sin \frac{50^\circ - 70^\circ}{2} + \sin 10^\circ$

$= 2 \cdot \cos 60^\circ \cdot \sin(-10^\circ) + \sin 10^\circ$

$= -2 \cdot \frac{1}{2} \cdot \sin 10^\circ + \sin 10^\circ$

$= 0$

10.



$\frac{\sin A + \sin B}{\cos A + \cos B} = 2 \cdot \cos \frac{C}{2}$ $\quad \angle C = ?$

sol) $\sin A + \sin B = 2 \cdot \sin \frac{A+B}{2} \cos \frac{A-B}{2}$

$\cos A + \cos B = 2 \cdot \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

$\therefore \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{2 \cdot \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2}}{2 \cdot \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2}} = \frac{\sin \frac{A+B}{2}}{\cos \frac{A+B}{2}}$

삼각형의 특성 : $A + B + C = \pi$

$A + B = \pi - C$

$\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$$\sin\frac{A+B}{2} = \sin\left(\frac{\pi}{2}-\frac{C}{2}\right) = \sin\frac{\pi}{2}\cos\frac{C}{2} - \cos\frac{\pi}{2}\sin\frac{C}{2}$$

$$= \cos\frac{C}{2} - 0$$

$$\cos\frac{A+B}{2} = \cos\left(\frac{\pi}{2}-\frac{C}{2}\right) = \cos\frac{\pi}{2}\cdot\cos\frac{C}{2} + \sin\frac{\pi}{2}\sin\frac{C}{2}$$

$$= 0 + \sin\frac{C}{2}$$

$$\therefore \frac{\sin\frac{A+B}{2}}{\cos\frac{A+B}{2}} = \frac{\cos\frac{C}{2}}{\sin\frac{C}{2}} = 2\cdot\cos\frac{C}{2}$$

$$\therefore \sin\frac{C}{2} = \frac{1}{2}$$

$$\frac{C}{2} = \frac{\pi}{6} \quad \left(\because\ 0<C<\pi \quad \therefore\ 0<\frac{C}{2}<\frac{\pi}{2}\right)$$

$$\therefore C = \frac{\pi}{3}$$

11.

① $\cos A + \cos B + \cos C = 1 + 4\cdot\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

$\cos A + \cos B + \cos C = 2\cdot\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2} + \cos C$ 합 → 곱

$\cos\frac{A+B}{2} = \sin\frac{C}{2}$ $\quad = 2\cdot\sin\frac{C}{2}\cdot\cos\frac{A-B}{2} + \cos 2\frac{C}{2}$ 배각공식.

$= 2\cdot\sin\frac{C}{2}\cdot\cos\frac{A-B}{2} + \cos^2\frac{C}{2} - \sin^2\frac{C}{2}$

$\sin^2\theta + \cos^2\theta = 1$ $\quad = 2\cdot\sin\frac{C}{2}\cdot\cos\frac{A-B}{2} + 1 - \sin^2\frac{C}{2} - \sin^2\frac{C}{2}$

$= 1 + 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\frac{C}{2}\right)$

$\sin\frac{C}{2} = \cos\frac{A+B}{2}$ $\quad = 1 + 2\cdot\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)$

$= 1 + 2\cdot\sin\frac{C}{2}\left(-2\cdot\sin\frac{A}{2}\sin\frac{-B}{2}\right)$ 합 → 곱

$= 1 + 2\cdot\sin\frac{C}{2}\left(2\cdot\sin\frac{A}{2}\cdot\sin\frac{B}{2}\right)$ $\sin(-\theta) = -\sin\theta$

$= 1 + 4\cdot\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

② $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

$$\tan A + \tan B + \tan C = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} + \frac{\sin C}{\cos C}$$

$$= \frac{\sin A \cdot \cos B \cdot \cos C + \cos A \cdot \sin B \cdot \cos C + \cos A \cdot \cos B \cdot \sin C}{\cos A \cdot \cos B \cdot \cos C}$$

$$\sin A \cdot \cos B \cdot \cos C + \cos A \cdot \sin B \cdot \cos C + \cos A \cdot \cos B \cdot \sin C$$

$$= (\sin A \cdot \cos B + \cos A \cdot \sin B)\cos C + \cos A \cdot \cos B \cdot \sin C$$

$$= \sin(A+B) \cdot \cos C + \frac{1}{2}\{\cos(A+B) + \cos(A-B)\}\sin C$$

$$= \sin(\pi - C) \cdot \cos C + \frac{1}{2}\{\cos(\pi - C) + \cos(A-B)\}\sin C$$

$$= (\cancel{\sin\pi \cdot \cos C}^{\,0} - \cos\pi \cdot \sin C)\cos C + \frac{1}{2}\{\cos\pi \cdot \cos C + \cancel{\sin\pi \cdot \sin C}^{\,0} + \cos(A-B)\}\sin C$$

$$= -(-1)\sin \cdot \cos C + \frac{1}{2}\{-\cos C + \cos(A-B)\}\sin C$$

$$= \sin C\left\{\cos C - \frac{1}{2}\cos C + \frac{1}{2}\cos(A-B)\right\}$$

$$= \sin C\left\{\frac{1}{2}\cos C + \frac{1}{2}\cos(A-B)\right\}$$

$$= \frac{1}{2} \cdot \sin C\left\{2 \cdot \cos\frac{C+A-B}{2} \cdot \cos\frac{C-A+B}{2}\right\}$$

$$= \sin C \cdot \cos\frac{\pi - 2B}{2} \cdot \cos\frac{\pi - 2A}{2}$$

$$= \sin C \cdot \left(\cancel{\cos\frac{\pi}{2}}^{\,0} \cdot \cos B + \sin\frac{\pi}{2} \cdot \sin B\right) \cdot \left(\cancel{\cos\frac{\pi}{2}}^{\,0} \cos A + \sin\frac{\pi}{2} \cdot \sin A\right)$$

$$= \sin A \cdot \sin B \cdot \sin C$$

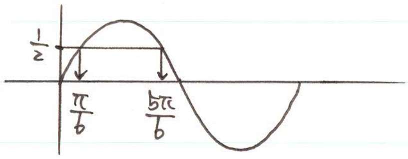
$$\therefore \tan A + \tan B + \tan C = \frac{\sin A \ \sin B \cdot \sin C}{\cos A \cdot \cos B \cdot \cos C} = \tan A \cdot \tan B \cdot \tan C$$

1.6 삼각방정식.

예제 1) $0 \le x < 2\pi$ 에서 삼각방정식 $f(x)=0$ 의 특수해를 구하라.

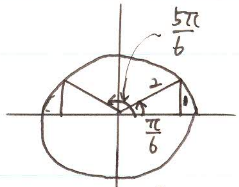
① $\sin x = \frac{1}{2}$

i)



또는

ii)

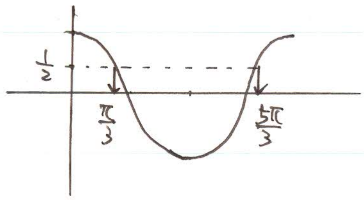


$\therefore x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

② $2\cos\left(x+\frac{\pi}{3}\right)=1$

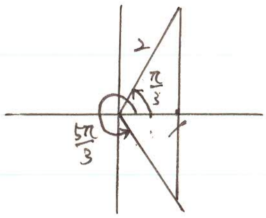
$\cos\left(x+\frac{\pi}{3}\right)=\frac{1}{2}$

i)



또는

ii)



$\therefore x+\frac{\pi}{3}=\frac{\pi}{3}$ or $x+\frac{\pi}{3}=\frac{5\pi}{3}$

$\therefore x=0$ or $x=\frac{4\pi}{3}$

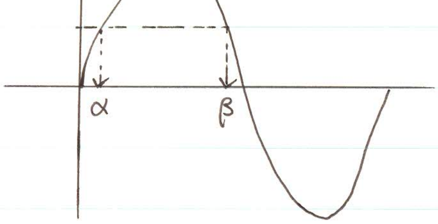
NOTE: $\cos x = \frac{1}{2}$ 에서 x는 $\frac{\pi}{3}$ or $\frac{5\pi}{3}$

$\cos\left(x+\frac{\pi}{3}\right)$는 $\cos x$를 x축으로 $-\frac{\pi}{3}$ 이동.

$\therefore \frac{\pi}{3}-\frac{\pi}{3}=0, \quad \frac{5\pi}{3}-\frac{\pi}{3}=\frac{4\pi}{3}$

NOTE: 삼각방정식의 일반해.

o $\sin x = a$ 의 해 α, β는



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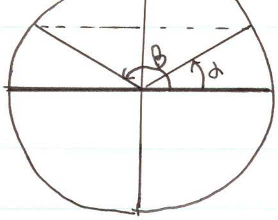

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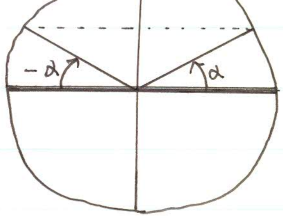
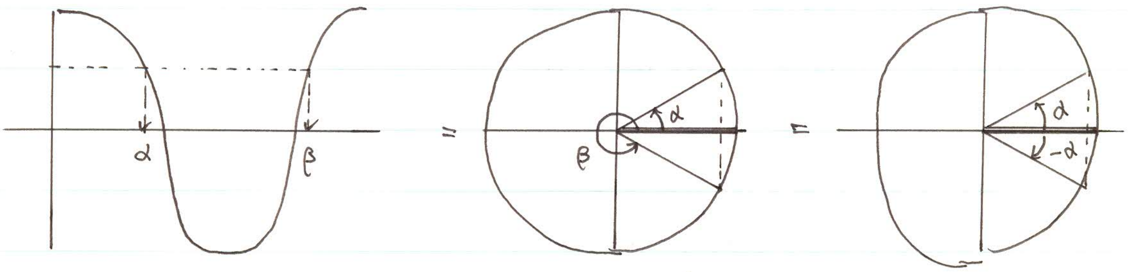


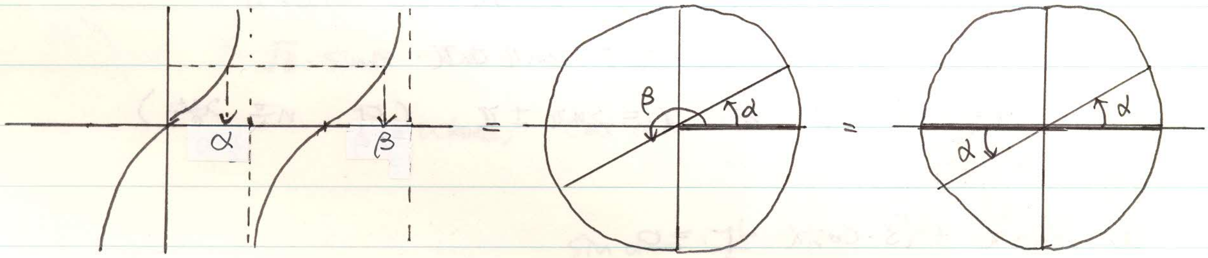
$$\left.\begin{array}{l} 2n\pi+\alpha \\ 2n\pi+\beta \end{array}\right\} = n\pi+(-1)^n\alpha$$

○ $\cos x = a$ 의 해 $\alpha \cdot \beta$ 는



$$\left.\begin{matrix} 2n\pi + \alpha \\ 2n\pi + \beta \end{matrix}\right\} = 2n\pi \pm \alpha$$

○ $\tan x = a$ 의 해 $\alpha \cdot \beta$ 는



$$\left.\begin{matrix} 2n\pi + \alpha \\ 2n\pi + \beta \end{matrix}\right\} = n\pi + \alpha$$

예제2) $\cos x - \sin 2x = 0$

$\cos x - 2\sin x \cdot \cos x = 0$

$\cos x (1 - 2 \cdot \sin x) = 0$

$\therefore \cos x = 0$ or $1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$

• $\cos x = 0$ 의 특수해 $x = \frac{\pi}{2}, \frac{3}{2}\pi$

$\therefore x = 2n\pi \pm \frac{\pi}{2}$

• $\sin x = \frac{1}{2}$ 의 특수해 $x = \frac{\pi}{6}, \frac{5}{6}\pi$

$\therefore x = n\pi + (-1)^n \frac{\pi}{6}$

$\therefore x = 2n\pi \pm \frac{\pi}{2}$ or $x = n\pi + (-1)^n \frac{\pi}{6}$ 단, n은 정수.

예제 3)

① $\cos 2x + \cos x = 0$

$\cos^2 x - \sin^2 x + \cos x = 0$

$\cos^2 x - (1 - \cos^2 x) + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\therefore \cos x = \frac{1}{2}$ or $\cos x = -1$

• $\cos x = \frac{1}{2}$의 특수해 $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore x = 2n\pi \pm \frac{\pi}{3}$

• $\cos x = -1$의 특수해 $x = \pi, 3\pi, \cdots$

$\therefore x = 2n\pi \pm \pi$

$\therefore x = 2n\pi \pm \frac{\pi}{3}$ or $x = 2n\pi \pm \pi$ (단, n은 정수)

② $\sin x + \sqrt{3} \cdot \cos x - 1 = 0$

• "삼각함수의 합성"을 이용하면

$$\sin x + \sqrt{3} \cdot \cos x = \sqrt{1+3} \cdot \sin(x + \alpha)$$

where, $\left.\begin{aligned} \cos\alpha &= \frac{1}{\sqrt{1+3}} = \frac{1}{2} \\ \sin\alpha &= \frac{\sqrt{3}}{\sqrt{1+3}} = \frac{\sqrt{3}}{2} \end{aligned}\right\} \therefore \alpha = \frac{\pi}{3}$

$\therefore \sin x + \sqrt{3} \cdot \cos x - 1 = 0$.

$2 \cdot \sin\left(x + \frac{\pi}{3}\right) - 1 = 0$

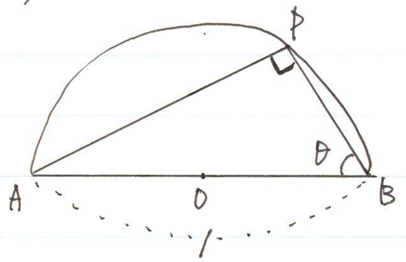
$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

$x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore x + \frac{\pi}{3} = n\pi + (-1)^n \cdot \frac{\pi}{6}$

$\therefore x = n\pi + (-1)^n \cdot \frac{\pi}{6} - \frac{\pi}{3}$ (단, n은 정수)

예제 4)



$\overline{BP} = \sqrt{3} \cdot \overline{AP} - \sqrt{2}$ 일 때 $\theta = ?$

$\angle APB = 90^\circ$ 이므로.

$\overline{BP} = 1 \cdot \cos\theta$

$\overline{AP} = 1 \cdot \sin\theta$

$\therefore \overline{BP} = \sqrt{3} \cdot \overline{AP} - \sqrt{2}$

$\cos\theta = \sqrt{3} \cdot \sin\theta - \sqrt{2}$

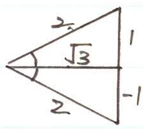
$\sqrt{3} \cdot \sin\theta - \cos\theta = \sqrt{2}$

"삼각함수의 합성" 을 이용하면

$\sqrt{3} \cdot \sin\theta - \cos\theta = \sqrt{3+1}\ \sin(\theta + \alpha)$

where, $\cos\alpha = \dfrac{\sqrt{3}}{2}$

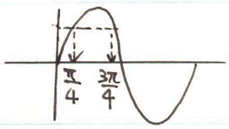
$\sin\alpha = \dfrac{-1}{2}$



$\therefore \alpha = -\dfrac{\pi}{6}$

$\therefore \sqrt{3} \cdot \sin\theta - \cos\theta = 2 \cdot \sin\left(\theta - \dfrac{\pi}{6}\right) = \sqrt{2}$

$\sin\left(\theta - \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{2}}$



$\theta - \dfrac{\pi}{6} = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$

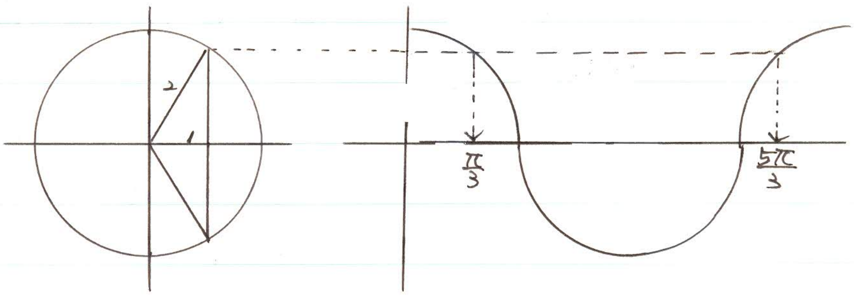
$\theta = \dfrac{\pi}{4} + \dfrac{\pi}{6}$ or $\dfrac{3\pi}{4} + \dfrac{\pi}{6}$

$\therefore \theta = \dfrac{5\pi}{12}$ or $\cancel{\dfrac{11\pi}{12}}$

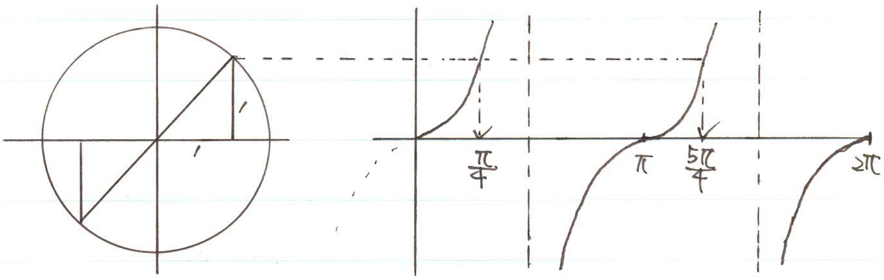
단, $\theta < \dfrac{\pi}{2}$ $\therefore \theta = \dfrac{5\pi}{12}$

1. 삼각방정식 그래프를 이용하여 풀어라 (단, $0 \leq x < 2\pi$)

① $\cos x = \frac{1}{2}$



② $\tan x = 1$



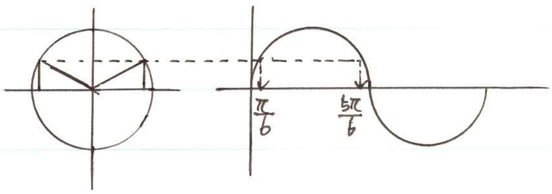
2 삼각방정식 단, $0 \leq x < 2\pi$

① $2\sin^2 x + 3\sin x - 2 = 0$

$(2\cdot\sin x - 1)(\sin x + 2) = 0$

$\therefore \sin x = \frac{1}{2}$ or $\cancel{\sin x = -2}$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$



② $2\cos^2 x + \sin x - 1 = 0$

$2(1 - \sin^2 x) + \sin x - 1 = 0$

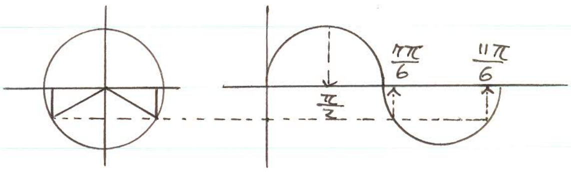
$2 - 2\sin^2 x + \sin x - 1 = 0$

$2\cdot\sin^2 x - \sin x - 1 = 0$

$(2\cdot\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$



3. 삼각방정식. 일반해.

① $\cos x + \sin 2x = 0$

$\cos x + 2\cdot \sin x \cdot \cos x = 0$

$\cos x\,(1 + 2\cdot \sin x) = 0$

$\cos x = 0$ or $\sin x = -\frac{1}{2}$

$$\left.\begin{array}{l} \cdot\ x = 2n\pi \pm \frac{\pi}{2} \\ \text{or}\ \cdot\ x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right) = n\pi - (-1)^n \frac{\pi}{6} \end{array}\right\} \text{단, } n\text{은 정수}$$

② $\cos 2x = 3\cos x + 1$

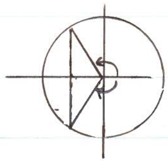
$\cos^2 x - \sin^2 x = 3\cos x + 1$

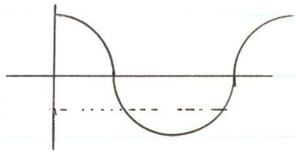
$\cos^2 x - (1 - \cos^2 x) = 3\cdot \cos x + 1$

$\cos^2 x - 1 + \cos^2 x = 3\cdot \cos x + 1$

$2\cdot \cos^2 x - 3\cos x - 2 = 0$

$(2\cdot \cos x + 1)(\cos x - 2) = 0$





$\cos x = -\frac{1}{2}$ or $\cancel{\cos x = 2}$

$\cdot\ x = 2n\pi \pm \frac{2\pi}{3}$ (단, n은 정수)

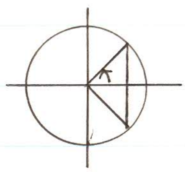
4. 삼각방정식.

① $\sin x + \cos x = 1$

"삼각함수의 합성"을 이용하면

$\sin x + \cos x = \sqrt{1+1}\,\sin(x + \alpha)$

$$\text{where,}\quad \left.\begin{array}{l} \cos\alpha = \frac{1}{\sqrt{2}} \\ \sin\alpha = \frac{1}{\sqrt{2}} \end{array}\right\} \therefore \alpha = \frac{\pi}{4}$$



$\therefore \sin x + \cos x = 1$

$\sqrt{2}\cdot \sin\left(x + \frac{\pi}{4}\right) = 1$

$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

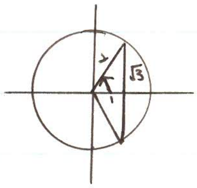
$\therefore x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$ (단, n은 정수)

② $\sqrt{3}\cos x + \sin x = -1$

"삼각함수의 합성"을 이용하면.

$\sin x + \sqrt{3}\cdot\cos x = \sqrt{1+3}\cdot\sin(x+\alpha)$

where, $\left.\begin{array}{l}\cos\alpha = \frac{1}{2}\\ \sin\alpha = \frac{\sqrt{3}}{2}\end{array}\right\}\ \alpha = \frac{\pi}{3}$



$\therefore\ \sqrt{3}\cdot\cos x + \sin x = -1$

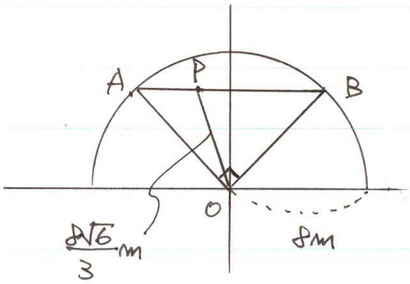
$2\cdot\sin(x+\frac{\pi}{3}) = -1$

$\sin(x+\frac{\pi}{3}) = -\frac{1}{2}$

$x+\frac{\pi}{3} = n\pi - (-1)^n\frac{\pi}{6}$

$\therefore\ x = n\pi - (-1)^n\frac{\pi}{6} - \frac{\pi}{3}$ (단, n은 정수)

5.

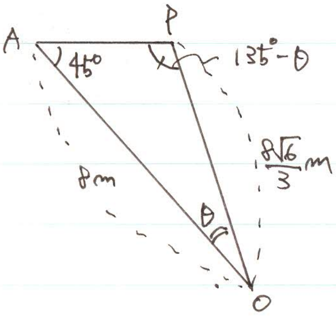


• ∠O는 90°이고, $\overline{AO}$, $\overline{BO}$는 반지름으로 8m.

∴ △AOB는 직각이등변삼각형,

∴ ∠A, ∠B는 45°

"Sine 법칙" $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$



$\frac{\frac{8\sqrt{6}}{3}}{\sin 45°} = \frac{8}{\sin(135-\theta)}$

$\frac{\frac{8\sqrt{6}}{3}}{\frac{1}{\sqrt{2}}} = \frac{16\sqrt{3}}{3} = \frac{8}{\sin(135-\theta)}$

$\sin(135-\theta) = \frac{\sqrt{3}}{2}$

$\underbrace{135-\theta}_{=\frac{3\pi}{4}} = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

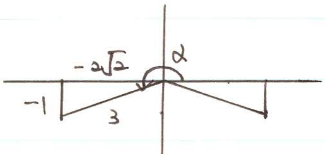
$\therefore\ \theta = \frac{3\pi}{4} - \frac{\pi}{3} = \frac{5\pi}{12}$ or $\theta = \frac{3\pi}{4} - \frac{2\pi}{3} = \frac{\pi}{12}$

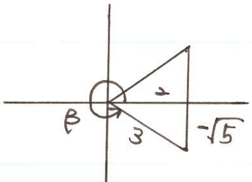
$\angle AOP = \frac{\pi}{12}$, $\angle BOP = \frac{5\pi}{12}$

1장 종합문제.

[1-A]

1. $\sin\alpha = -\frac{1}{3}$, $\cos\beta = \frac{2}{3}$, $\cos\alpha, \sin\beta$는 (−)일때. 다음을 구하라.





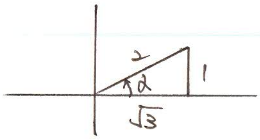
$\sin\alpha\,(-)$, $\cos\alpha\,(-)$ $\qquad$ $\cos\beta\,(+)$, $\sin\beta\,(-)$

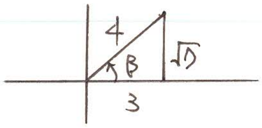
∴ 3 사분면 $\quad \cos\alpha = \frac{-2\sqrt{2}}{3}$ $\qquad$ ∴ 4 사분면 $\quad \sin\beta = \frac{-\sqrt{5}}{3}$

① $\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$
$$= \left(-\frac{1}{3}\right)\frac{2}{3} + \left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(-\frac{\sqrt{5}}{3}\right)$$
$$= -\frac{2}{9} + \frac{2\sqrt{10}}{9}$$
$$= \frac{2}{9}\left(\sqrt{10}-1\right)$$

② $\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$
$$= \left(-\frac{2\sqrt{2}}{3}\right)\frac{2}{3} + \left(-\frac{1}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$$
$$= -\frac{4\sqrt{2}}{9} + \frac{\sqrt{5}}{9}$$
$$= \frac{1}{9}\left(\sqrt{5}-4\sqrt{2}\right)$$

2. α, β가 모두 예각이고, $\sin\alpha = \frac{1}{2}$, $\cos\beta = \frac{3}{4}$ 일 때 다음을 구하라.





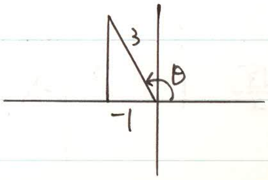
∴ $\cos\alpha = \frac{\sqrt{3}}{2}$, $\tan\alpha = \frac{1}{\sqrt{3}}$ $\qquad \sin\beta = \frac{\sqrt{7}}{4}$

① $\sin 2\beta = 2\cdot\sin\beta\cdot\cos\beta = 2\times\frac{\sqrt{7}}{4}\cdot\frac{3}{4} = \frac{3\sqrt{7}}{8}$

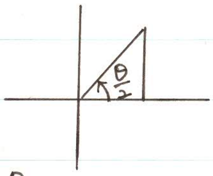
② $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

③ $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\cdot\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$

3. $\cos\theta = -\frac{1}{3}$ (단, $\frac{\pi}{2} < \theta < \pi$) 일 때, 다음을 구하라.



$\Rightarrow \frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$



θ는 2사분면

$\frac{\theta}{2}$는 1사분면 각.

① $\sin\frac{\theta}{2} = ?$

$$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2} = \frac{1-(-\frac{1}{3})}{2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

$\therefore \sin\frac{\theta}{2} = \pm\sqrt{\frac{2}{3}}$

$\Rightarrow$ 1사분면각 $\sin\frac{\theta}{2}$는 $(+)$ $\therefore \sin\frac{\theta}{2} = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$.

② $\cos\frac{\theta}{2} = ?$

$$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2} = \frac{1+(-\frac{1}{3})}{2} = \frac{\frac{2}{3}}{2} = \frac{1}{3}$$

$\therefore \cos\frac{\theta}{2} = \pm\sqrt{\frac{1}{3}}$

$\Rightarrow$ 1사분면각 $\cos\frac{\theta}{2}$는 $(+)$ $\therefore \cos\frac{\theta}{2} = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3}$

③ $\tan\frac{\theta}{2} = ?$

$$\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-(-\frac{1}{3})}{1+(-\frac{1}{3})} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2.$$

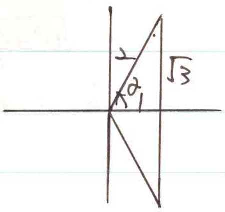
$\therefore \tan\frac{\theta}{2} = \pm\sqrt{2}$

$\Rightarrow$ 1사분면각 $\tan\frac{\theta}{2}$는 $(+)$ $\therefore \tan\frac{\theta}{2} = \sqrt{2}$

4 "삼각함수 합성" 하라.

① $\sin\theta + \sqrt{3}\cos\theta = \sqrt{1+3}\,\sin(\theta+\alpha)$

where, $\left.\begin{array}{l}\cos\alpha = \frac{1}{2}\\ \sin\alpha = \frac{\sqrt{3}}{2}\end{array}\right\}$



$\alpha = \frac{\pi}{3}$

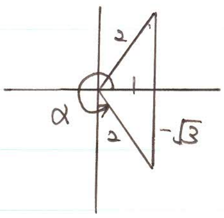
$\therefore \sin\theta + \sqrt{3}\cdot\cos\theta$

$= 2\cdot\sin(\theta+\frac{\pi}{3})$

② $2\cdot\sin\theta - 2\sqrt{3}\cdot\cos\theta = \sqrt{4+12}\,\sin(\theta+\alpha)$

where, $\cos\alpha = \frac{2}{4} = \frac{1}{2}$

$\sin\alpha = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$



$\therefore \alpha = -\frac{\pi}{3}$

$\therefore 2\cdot\sin\theta - 2\sqrt{3}\cdot\cos\theta$

$= 4\cdot\sin\left(\theta - \frac{\pi}{3}\right)$

5. $f(x) = a\cdot\sin x + 2\cdot\cos x + b$, $\{f(x) \mid 1 \leqq f(x) \leqq 6\}$.

$a+b=?$ (단, $a>0$)

sol) $a\cdot\sin x + 2\cdot\cos x = \sqrt{a^2+4}\,\sin(x+\alpha)$

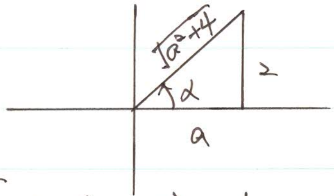
where, $\cos\alpha = \frac{a}{\sqrt{a^2+4}}$

$\sin\alpha = \frac{2}{\sqrt{a^2+4}}$

$a>0$ 이므로 $\frac{a}{\sqrt{a^2+4}} > 0$ $\therefore$ α는 1사분면 or 4사분면.

〃 $\frac{2}{\sqrt{a^2+4}} > 0$ $\therefore$ α는 1사분면 or 2사분면

$\therefore$ α는 1사분면의 각



$\therefore f(x) = \sqrt{a^2+4}\,\sin(x+\alpha) + b$

$f(x).\min = -\sqrt{a^2+4} + b = 1$ — ①

$f(x).\max = \sqrt{a^2+4} + b = 6$ — ②

①에서 $b-1 = \sqrt{a^2+4}$. — ③

③ → ② $b-1+b = 2b-1 = 6$

$b = \frac{7}{2}$

③에서 $a^2+4 = \left(\frac{7}{2}-1\right)^2 = \frac{25}{4}$

$$a^2 = \frac{25}{4} - 4 = \frac{9}{4}$$

$$a = \frac{3}{2} \quad (\because a > 0)$$

$$\therefore a + b = \frac{3}{2} + \frac{7}{2} = \frac{10}{2} = 5$$

6. $\tan\frac{x}{2} = t$ 라고 할 때 다음을 증명하라.

① $\sin x = \frac{2t}{1+t^2}$

$$\sin x = \sin 2\cdot\frac{x}{2} = 2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2} = \frac{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}} = \frac{2\cdot\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

분자·분모를 $\cos^2\frac{x}{2}$ 으로 나누면

② $\cos x = \frac{1-t}{1+t}$

$$\cos x = \cos 2\cdot\frac{x}{2} = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}} = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

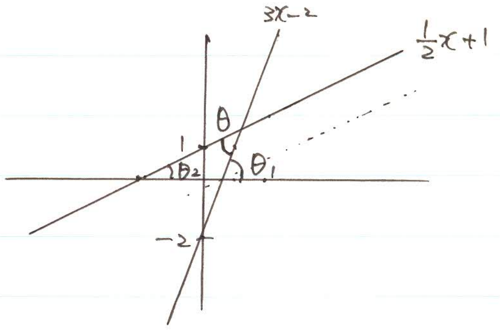
분자·분모를 $\cos^2\frac{x}{2}$ 으로 나누면

③ $\tan x = \frac{2t}{1-t^2}$

$$\tan x = \tan 2\cdot\frac{x}{2} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

7. 두 직선이 이루는 예각의 크기 θ

$$\left.\begin{array}{l} 3x - y - 2 = 0 \\ x - 2y + 2 = 0 \end{array}\right| \quad \begin{array}{l} y = 3x - 2 \\ y = \frac{1}{2}x + 1 \end{array}$$



그림에서 $\theta = \theta_1 - \theta_2$

$$\tan\theta = \tan(\theta_1 - \theta_2)$$
$$= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \cdot \tan\theta_2}$$

where, $\tan\theta_1 = 3$

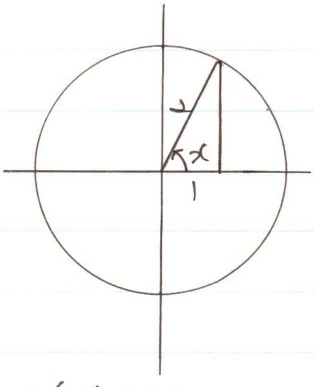
$\tan\theta_2 = \frac{1}{2}$

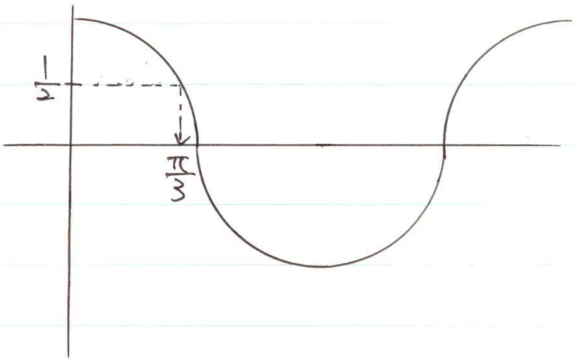
$$\therefore \tan\theta = \frac{3 - \frac{1}{2}}{1 + 3 \cdot \frac{1}{2}} = \frac{\frac{5}{2}}{\frac{5}{2}} = 1$$

$$\theta = \frac{\pi}{4}$$

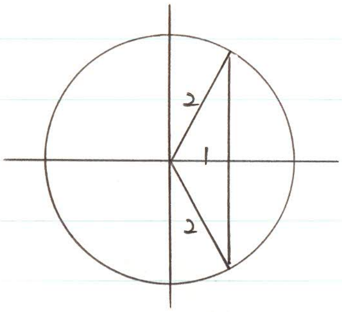
8. 주어진 범위에서 삼각방정식 $\cos x = \frac{1}{2}$의 해 구하기

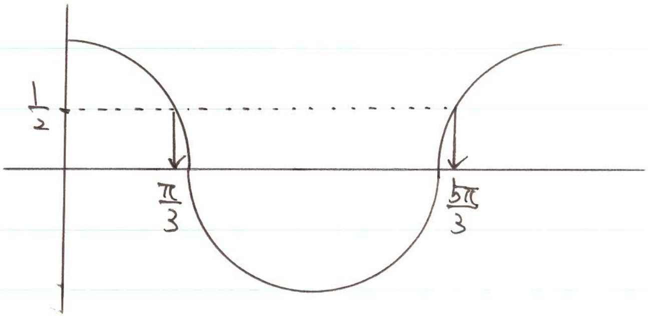
① $0 \leqq x < \frac{\pi}{2}$





② $0 \leqq x < 2\pi$





9. 삼각방정식.

① $\sin x = 1$

$$x = n\pi + (-1)^n \frac{\pi}{2}$$

$$= 2n\pi + \frac{\pi}{2} \quad (\text{단, } n\text{은 정수})$$

② $\cos x = \frac{\sqrt{3}}{2}$

$$x = 2n\pi \pm \frac{\pi}{6} \quad (\text{단, } n\text{은 정수})$$

③ $\tan x = \sqrt{3}$

$$x = n\pi + \frac{\pi}{3} \quad (\text{단, } n\text{은 정수})$$

[1-B]

1 다음의 값을 구하라.

① $\sin 15° + \cos 15° = \sqrt{2}\sin(15° + \alpha)$ "삼각함수의 합성"

$$\left.\begin{aligned} \text{where, } \cos\alpha &= \frac{1}{\sqrt{2}} \\ \sin\alpha &= \frac{1}{\sqrt{2}} \end{aligned}\right\} \therefore \alpha = \frac{\pi}{4} \sim 45°$$

$$\therefore \sin 15° + \cos 15° = \sqrt{2}\cdot\sin(15° + 45°)$$

$$= \sqrt{2}\cdot\sin 60°$$

$$= \sqrt{2}\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{6}}{2}$$

② $\sin 80° - \sin 40° - \sin 20°$ "합, 차를 곱으로"

$$= 2\cdot\cos\frac{80°+40°}{2}\cdot\sin\frac{80°-40°}{2} - \sin 20°$$

$$= 2\cdot\cos 60°\cdot\sin 20° - \sin 20°$$

$$= 2\cdot\frac{1}{2}\cdot\sin 20° - \sin 20°$$

$$= 0$$

2. 곱 → 합 or 차.

① $\sin 45^\circ \cdot \cos 15^\circ = \frac{1}{2}\{\sin(45^\circ+15^\circ) + \sin(45^\circ-15^\circ)\}$

$= \frac{1}{2}\{\sin 60^\circ + \sin 30^\circ\}$

$= \frac{1}{2}\{\frac{\sqrt{3}}{2} + \frac{1}{2}\}$

$= \frac{1}{4}(\sqrt{3}+1)$

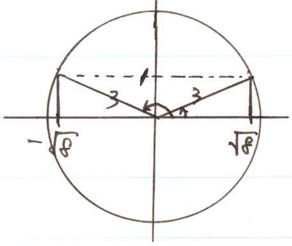
② $\cos 75^\circ \cdot \cos 15^\circ = \frac{1}{2}\{\cos(75^\circ+15^\circ) + \cos(75^\circ-15^\circ)\}$

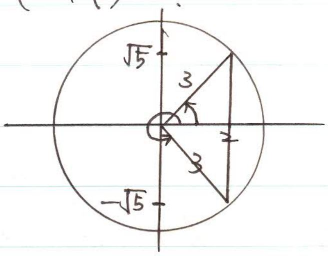
$= \frac{1}{2}\{\cos 90^\circ + \cos 60^\circ\}$

$= \frac{1}{2}\{0 + \frac{1}{2}\}$

$= \frac{1}{4}$

3. $\sin\alpha = \frac{1}{3}$, $\cos\beta = \frac{2}{3}$ 일 때 $\cos(\alpha+\beta) = ?$





$\therefore \cos\alpha = \frac{\sqrt{8}}{3}$ or $-\frac{\sqrt{8}}{3}$ $\qquad \sin\beta = \frac{\sqrt{5}}{3}$ or $-\frac{\sqrt{5}}{3}$

$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta.$

$\therefore \cos\alpha(+), \sin\beta(+) \to \cos(\alpha+\beta) = \frac{\sqrt{8}}{3}\cdot\frac{2}{3} - \frac{1}{3}\cdot\frac{\sqrt{5}}{3}$

$= \frac{4\sqrt{2}}{9} - \frac{\sqrt{5}}{9} = \frac{1}{9}(4\sqrt{2}-\sqrt{5})$

$\cos\alpha(+), \sin\beta(-) \to \cos(\alpha+\beta) = \frac{\sqrt{8}}{3}\cdot\frac{2}{3} - \frac{1}{3}\left(-\frac{\sqrt{5}}{3}\right)$

$= \frac{4\sqrt{2}}{9} + \frac{\sqrt{5}}{9} = \frac{1}{9}(4\sqrt{2}+\sqrt{5})$

$\cos\alpha(-), \sin\beta(+) \to \cos(\alpha+\beta) = \left(-\frac{\sqrt{8}}{3}\right)\frac{2}{3} - \frac{1}{3}\frac{\sqrt{5}}{3}$

$= -\frac{4\sqrt{2}}{9} - \frac{\sqrt{5}}{9} = -\frac{1}{9}(4\sqrt{2}+\sqrt{5})$

$\cos\alpha(-), \sin\beta(-) \to \cos(\alpha+\beta) = \left(-\frac{\sqrt{8}}{3}\right)\frac{2}{3} - \frac{1}{3}\left(-\frac{\sqrt{5}}{3}\right)$

$= -\frac{4\sqrt{2}}{9} + \frac{\sqrt{5}}{9} = -\frac{1}{9}(4\sqrt{2}-\sqrt{5})$

4. x에 대한 이차방정식 $x^2 - x\cdot\cos A + \sin A = 0$ 의 두 근은
$\tan\alpha$, $\tan\beta$ 이다, $\tan(\alpha+\beta) = \frac{1}{2}$ 일 때 $\sin A = ?$

$x = \tan\alpha \rightarrow \quad \tan^2\alpha - \tan\alpha\cdot\cos A + \sin A = 0 \quad$ —①

$x = \tan\beta \rightarrow \quad \tan^2\beta - \tan\beta\cdot\cos A + \sin A = 0 \quad$ —②

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta} = \frac{1}{2} \quad \text{—③}$$

① − ②

$$\begin{array}{rl} & \tan^2\alpha - \tan\alpha\cdot\cos A + \sin A = 0 \\ -) & \tan^2\beta - \tan\beta\cdot\cos A + \sin A = 0 \\ \hline \end{array}$$

$\tan^2\alpha - \tan^2\beta - \tan\alpha\cos A + \tan\beta\cdot\cos A = 0$

$(\tan\alpha + \tan\beta)(\tan\alpha - \tan\beta) - (\tan\alpha - \tan\beta)\cdot\cos A = 0$

$(\tan\alpha + \tan\beta - \cos A)(\tan\alpha - \tan\beta) = 0$

$\therefore \cos A = \tan\alpha + \tan\beta \quad$ —④

①$\cdot\tan\beta$ − ②$\cdot\tan\alpha$

$$\begin{array}{rl} & \tan^2\alpha\cdot\tan\beta - \tan\alpha\cdot\tan\beta\cdot\cos A + \tan\beta\cdot\sin A = 0 \\ -) & \tan\alpha\cdot\tan^2\beta - \tan\alpha\cdot\tan\beta\cdot\cos A + \tan\alpha\cdot\sin A = 0 \\ \hline \end{array}$$

$\tan^2\alpha\cdot\tan\beta - \tan\alpha\cdot\tan^2\beta + \tan\beta\cdot\sin A - \tan\alpha\cdot\sin A = 0$

$(\tan\alpha - \tan\beta)\tan\alpha\cdot\tan\beta - (\tan\alpha - \tan\beta)\sin A = 0$

$(\tan\alpha - \tan\beta)(\tan\alpha\cdot\tan\beta - \sin A) = 0$

$\therefore \sin A = \tan\alpha\cdot\tan\beta \quad$ —⑤

④.⑤ → ③

$$\tan(\alpha+\beta) = \frac{\cos A}{1 - \sin A} = \frac{1}{2} \quad \text{—⑥}$$

$2\cdot\cos A = 1 - \sin A$

$4\cdot\cos^2 A = 1 - 2\sin A + \sin^2 A$

$4(1 - \sin^2 A) = 4 - 4\cdot\sin^2 A = 1 - 2\cdot\sin A + \sin^2 A$

$5\cdot\sin^2 A - 2\cdot\sin A - 3 = 0$

$(5\cdot\sin A + 3)(\sin A - 1) = 0$

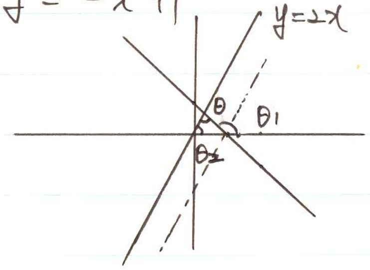
$\sin A = -\frac{3}{5}$ or 1

⑥에서 $\sin A \neq 1 \quad \therefore \sin A = -\frac{3}{5}$

5. 두 직선이 이루는 예각 θ에 대하여 $\tan\theta$ 구하기

$y = 2x$

$y = -x + 1$



$$\theta = \theta_1 - \theta_2$$

$$\tan\theta = \tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \cdot \tan\theta_2}$$

where, $\tan\theta_1 = -1$

$\tan\theta_2 = 2$

$$\therefore \tan\theta = \frac{-1-2}{1+(-1)\cdot 2} = \frac{-3}{-1} = 3$$

6. 최대 · 최소값 구하기.

$$f(x) = \cos^2 x - 2\cdot\cos x\cdot\sin x - 3\cdot\sin^2 x$$

$$= \frac{1+\cos 2x}{2} - \sin 2x - 3\cdot\frac{1-\cos 2x}{2}$$ "반각의 공식"

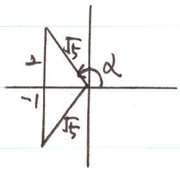
$$= \frac{1}{2} + \frac{1}{2}\cos 2x - \sin 2x - \frac{3}{2} + \frac{3}{2}\cos 2x$$

$$= -\sin 2x + 2\cdot\cos 2x - 1$$

$$= \sqrt{1+4}\cdot\sin(2x+\alpha) - 1$$ "삼각함수의 합성"

where, $\cos\alpha = \frac{-1}{\sqrt{5}}$

$\sin\alpha = \frac{2}{\sqrt{5}}$



$\therefore f(x).\max = \sqrt{5} - 1$

$f(x).\min = -\sqrt{5} - 1$

7. 코사인의 덧셈정리를 이용하여 다음을 보여라.

① $\cos\alpha \cdot \cos\beta = \frac{1}{2}\{\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$

$= \frac{1}{2}\{\cos\alpha \cdot \cos\beta - \cancel{\sin\alpha \cdot \sin\beta} + \cos\alpha \cdot \cos\beta + \cancel{\sin\alpha \cdot \sin\beta}\}$

$= \frac{1}{2}\{2 \cdot \cos\alpha \cdot \cos\beta\}$

$= \cos\alpha \cdot \cos\beta$.

② $\sin\alpha \cdot \sin\beta = -\frac{1}{2}\{\cos(\alpha+\beta) - \cos(\alpha-\beta)\}$

$= -\frac{1}{2}\{\cancel{\cos\alpha \cdot \cos\beta} - \sin\alpha \cdot \sin\beta - \cancel{\cos\alpha \cdot \cos\beta} - \sin\alpha \cdot \sin\beta\}$

$= -\frac{1}{2}\{-2 \cdot \sin\alpha \cdot \sin\beta\}$

$= \sin\alpha \cdot \sin\beta$.

8. 다음의 조건을 만족하는 삼각형은?

① $\sin(A+B) \cdot \sin(A-B) = \sin^2 C$

곱 → 합 or 차

$\sin(A+B) \cdot \sin(A-B) = -\frac{1}{2}\{\cos(A+B+A-B) - \cos(A+B-A+B)\}$

$= -\frac{1}{2}\{\cos(2A) - \cos(2B)\}$

"반각의 공식" $\sin^2 x = \frac{1-\cos 2x}{2}$

$= -\frac{1}{2}\{1 - 2 \cdot \sin^2 A - 1 + 2 \cdot \sin^2 B\}$

$= -\frac{1}{2}\{-2 \cdot \sin^2 A + 2 \cdot \sin^2 B\}$

$= \sin^2 A - \sin^2 B = \sin^2 C$

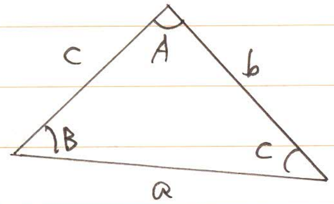
$\therefore \sin^2 A = \sin^2 B + \sin^2 C$

"사인의 법칙" $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$\therefore \sin^2 A = \frac{a^2}{4R^2}, \quad \sin^2 B = \frac{b^2}{4R^2} \quad \sin^2 C = \frac{c^2}{4R^2}$

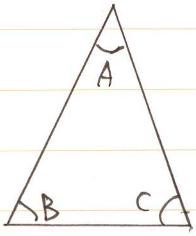
$\frac{a^2}{4R^2} = \frac{b^2}{4R^2} + \frac{c^2}{4R^2}$

$\therefore a^2 = b^2 + c^2$



$\therefore \angle A = 90^\circ$ 인 직각삼각형.

② $\cos A = 1 - 2\cdot\cos B\cdot\cos C$



$$A + B + C = \pi$$
$$B + C = \pi - A$$
$$\cos(B+C) = \cos(\pi - A)$$
$$\cos B\cdot\cos C - \sin B\cdot\sin C = -\cos A$$
$$= 2\cdot\cos B\cdot\cos C - 1$$
$$\therefore \cos B\cdot\cos C + \sin B\cdot\sin C = 1$$
$$\cos(B - C) = 1$$
$$B - C = 0 \quad \text{or} \quad \cancel{2\pi}$$
$$\therefore B = C$$

$\therefore \angle B = \angle C$ 인 이등변 삼각형.

9. 삼각방정식.

① $\cos 2x - \cos x + 1 = 0$

$$\cos^2 x - \sin^2 x - \cos x + 1 = 0$$
$$\cos^2 x - (1 - \cos^2 x) - \cos x + 1 = 0$$
$$\cos^2 x - 1 + \cos^2 x - \cos x + 1 = 0$$
$$2\cdot\cos^2 x - \cos x = 0$$
$$\cos x\,(2\cos x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \frac{1}{2}$$

$\therefore x = 2n\pi \pm \frac{\pi}{2}$ or $2n\pi \pm \frac{\pi}{3}$ (단, n은 정수)

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⑦ $\sin x + \sin 3x + \sin 5x = 0$

$(\sin 5x + \sin x) + \sin 3x = 0$

$2 \cdot \sin \frac{5x + x}{2} \cdot \cos \frac{5x - x}{2} + \sin 3x = 0$

$2 \cdot \sin 3x \cdot \cos 2x + \sin 3x = 0$

$\sin 3x \left(2 \cdot \cos 2x + 1 \right) = 0$

$\sin 3x = 0$ or $\cos 2x = -\frac{1}{2}$

$\therefore 3x = n\pi$ or $2x = 2n\pi \pm \frac{2\pi}{3}$

$\therefore x = \frac{1}{3}n\pi$ or $x = n\pi \pm \frac{\pi}{3}$ (단, n은 정수)